Original scientific papers UDK 620.1/.2:51−7
doi: 10.7251/COMEN1702128D

FINITE DIFFERENCE SOLUTION OF TWO-DIMENSIONAL SOLUTE TRANSPORT WITH PERIODIC FLOW IN HOMOGENOUS POROUS MEDIA

Alexandar Djordjevich[1], Svetislav Savović[2], Aco Janićijević[3]*
[1] City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong, China
[2] University of Kragujevac, Faculty of Science, R. Domanovića 12, Kragujevac, Serbia
[3] University of Belgrade, Faculty of Technology and Metalurgy, Karnegijeva 4, Belgrade, Serbia

Abstract: Two-dimensional advection-diffusion equation with variable coefficients is solved by the explicit finite-difference method for the transport of solutes through a homogeneous, finite, porous, two-dimensional, domain. Retardation by adsorption, periodic seepage velocity, and a dispersion coefficient proportional to this velocity are permitted. The transport is from a pulse-type point source (that ceases after a period of activity). Included are the first-order decay and zero-order production parameters proportional to the seepage velocity, periodic boundary conditions at the origin and the end of the domain. Results are compared to analytical solutions reported in the literature for special cases and a good agreement was found. The solute concentration profile is greatly influenced by the periodic velocity fluctuations. Solutions for a variety of combinations of unsteadiness of the coefficients in the advection-diffusion equation are obtainable as particular cases of the one demonstrated here. This further attests to the effectiveness of the explicit finite difference method for solving two-dimensional advection-diffusion equation with variable coefficients in a finite media, which is especially important when arbitrary initial and boundary conditions are required.

Keywords: two dimensional advection-diffusion equation; mass transfer; finite difference method.

1. INTRODUCTION

Much of the research interest in the problem of solute transport through porous media stems from concerns for the degradation of air, soil, surface water and groundwater. Many factors affect this transport including the solvent and solute properties, fluid velocity field within the porous medium and micro-geometry such as the shape, size, and location of the solid part of the medium or the layout of the voids. Advective-dispersive phenomena often dominate the process. For groundwater, contaminants permeate through pores in the ground. Adsorption attenuates the resulting pollution concentration as the pollutants adhere to the solid surface.

The partial differential advection–diffusion equation describes the solute transport through a porous medium in problems of biophysics, soil physics, chemical engineering and petroleum engineering, among other fields. For example, petroleum engineers may use this equation to model multiphase and multicomponent flows in the extraction of hydrocarbons from petroleum reservoirs and to predict the concentration of toxic, reactive or radioactive pollutants from mining operations that may pose adverse potential for the downstream flora and fauna [1]. Such studies play an important role in assessing risks and devising remedial environmental management.

Lindstrom and Boersma [2] have reviewed analytical solutions describing the solute transport through one-dimensional media while accounting for adsorption, first-order decay and zero-order production. To address fluctuations of the groundwater table and flow patterns caused by the periodicity of the sea level, Logan and Zlotnik [3] proposed analytical solutions with a decay term for periodic input conditions through a semi-infinite domain. Townley [4] obtained analytical solutions for a periodic aquifer flow. Jaiswal

* Corresponding author: savovic@kg.ac.rs

et al. [5,6], Kumar et al. [7], and Yadav et al. [8] obtained analytical solutions for one-dimensional advection–diffusion equation with temporally and spatially dependent dispersion problems.

Numerical solutions of advection-diffusion equation have also been reported for more general engineering problems not covered by analytical solutions, especially two- and three-dimensional problems [9−17]. We have demonstrated in our recent works [18−21] that explicit finite difference method (EFDM) is effective and accurate in solving one-dimensional advection-diffusion equation with variable coefficients, as well as in solving two-dimensional advection-diffusion equation for solute transport from a pulse-type source (that ceases after a brief activity) along temporally and spatially dependent flow [21]. In this study, we derive a numerical solution for transport of solutes from a point source through a homogeneous, finite, porous, two-dimensional domain. The solution allows for (i) periodic seepage velocity and (ii) dispersion coefficient that is proportional to the seepage velocity. The retardation process that occurs in the porous medium due to adsorption is also taken into account. The solute transport is assumed to be against the longitudinal direction of groundwater velocity [22,23].

Such situations often occur in practice when poor-quality water is prevented from spreading by a flow of fresh water. Mixed-type boundary condition at the origin of the domain and boundary condition of periodic nature at the end of the domain are considered. The solution is illustrated to demonstrate the solute transport, both along the longitudinal and transverse directions. A significant transverse solute transport has been noted even for very low longitudinal velocity and dispersivity relative to the respective values in the longitudinal direction. Because one-dimensional models cannot capture such transverse transport, a two-dimensional model is required. Moreover, for different combinations of unsteadiness of the coefficients in the advection-diffusion equation, solutions can be obtained as particular cases of the one obtained in the present study.

2. ADVECTION-DIFFUSION EQUATION

Let the polluting solute particles enter a porous medium at a fixed location, continuously and at a constant rate up to a certain moment when the flow ceases (pulse-type source). The advection–diffusion equation in two-dimensional horizontal plane medium may be written as [24]:

$$R_f \frac{\partial C(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(D_x(x,t)\frac{\partial C(x,y,t)}{\partial x} - u(x,t)C(x,y,t)\right) + \frac{\partial}{\partial y}\left(D_y(y,t)\frac{\partial C(x,y,t)}{\partial y} - v(y,t)C(x,y,t)\right) - \gamma(x,y,t)C + \mu(x,y,t) \quad (1)$$

where $C(x,y,t)$ is the dispersing solute concentration at a position (x,y) at time t; $D_x(x,t)$ and $D_y(y,t)$ are dispersion coefficients in directions x and y, respectively; $u(x,t)$ and $v(y,t)$ are velocity coefficients in directions x and y, respectively; γ is the first-order decay coefficient and μ is the zero-order production coefficient. The retardation factor $R_f = \left(1 + \frac{1-n_p}{n_p}K_1\right)$ accounts for the equilibrium linear adsorption process, where n_p is the porosity of the medium and K_1 is an empirical constant [25,26].

The expressions for velocity components are assumed in the following form [22]:

$$u(x,t) = u_0\left|\sin(mt)\right|; \quad v(y,t) = v_0\left|\sin(mt)\right| \quad (2)$$

where u_0 and v_0 are uniform longitudinal and transverse velocity components, respectively, each of dimension (LT^{-1}). The coefficient m represents the unsteadiness parameter. It is of dimension inverse of time. The inclusion of transverse diffusion makes the dispersion problem two-dimensional. Hence, we consider dispersion coefficients as:

$$D(x,t) = D_{x_0}\left|\sin(mt)\right|, \quad D(y,t) = D_{y_0}\left|\sin(mt)\right| \quad (3)$$

where D_{x_0} and D_{y_0} are the initial longitudinal and transverse dispersion coefficients, respectively, each of dimension (L^2T^{-1}). Furthermore, first-order decay and zero-order production are considered directly proportional to the seepage velocity and, thus, it is:

$$\gamma = \gamma_0\left|\sin(mt)\right|, \quad \mu = \mu_0\left|\sin(mt)\right| \quad (4)$$

where γ_0 and μ_0 are constants of dimension T^{-1} and $ML^{-3}T^{-1}$, respectively. Hence, (1) can now be rewritten as:

$$R_f \frac{\partial C(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(D_{x_0}\left|\sin(mt)\right|\frac{\partial C(x,y,t)}{\partial x} - u_0\left|\sin(mt)\right|C(x,y,t)\right) + \frac{\partial}{\partial y}\left(D_{y_0}\left|\sin(mt)\right|\frac{\partial C(x,y,t)}{\partial y} - v_0\left|\sin(mt)\right|C(x,y,t)\right) - \gamma_0\left|\sin(mt)\right|C + \mu_0\left|\sin(mt)\right| \quad (5)$$

For the plane and horizontal porous medium, the initial solute concentration is set to vary with position according to (6) whereby x-axis is longitudinal and y-axis is transverse to the flow. The solute transport is in finite space $L_{x_1} \le x \le L_x$ and $L_{y_1} \le y \le L_y$ in the x>0 and y>0 directions. The source of the pollution is considered to be a uniform pulse at point (L_x, L_y). In other words, the pollution dispersion exists against the flow. Let the time of elimination of the point source be t_0. The initial and boundary conditions are [24]:

$$C(x,y,t) = C_{in}\exp(\alpha x + \alpha y),\ L_{x_1} \le x \le L_x; L_{y_1} \le y \le L_y;\ t=0 \tag{6}$$

$$C(x,y,t) = \begin{cases} C_0(1+\cos(mt)), & 0 < t \le t_0;\ x = L_x, y = L_y \\ 0, & t > t_0;\quad x = L_x, y = L_y \end{cases} \tag{7}$$

$$\frac{\partial C(x,y,t)}{\partial x} = \frac{u(x,t)}{2D_x(x,t)}C(x,y,t),\ x = L_{x_1};\ \frac{\partial C(x,y,t)}{\partial y} = \frac{v(y,t)}{2D_y(y,t)}C(x,y,t),\quad y \to L_{y_1};\ (t \ge 0) \tag{8}$$

where C_0 is the reference concentration representing the input concentration that is released uniformly by the source and α is a constant whose dimension is the inverse of length.

3. ANALYTICAL SOLUTION OF ADVECTION-DIFFUSION EQUATION

Analytical solution of the advection-diffusion equation (1), subject to initial condition (6) and boundary conditions (7) and (8), is [24]:

$$C(\eta,T) = F_1(\eta,T) + C_{in}F_2(\eta,T) + \left(2C_0 - \frac{\mu_0}{\gamma_0}\right)F_3(\eta,T) + C_0F_4(\eta,T);\quad 0 < T \le T_0 \tag{9}$$

$$\begin{aligned} C(\eta,T) &= F_1(\eta,T) + C_{in}F_2(\eta,T) + C_0\left[2F_3(\eta,T) - (2 - mR_fT_0)F_3(\eta,T-T_0)\right] \\ &+ C_0\left[F_4(\eta,T) - F_4(\eta,T-T_0)\right]; \qquad T > T_0 \end{aligned} \tag{10}$$

Where

$$\begin{aligned} F_1(\eta,T) &= \frac{\mu_0}{\gamma_0} - \frac{\mu_0}{\gamma_0}\exp(-\gamma_0T) + \frac{\mu_0}{\gamma_0}\exp(-\beta L + \beta\eta)\left[\exp(-\gamma_0T)\frac{\cosh(\eta-L)\beta}{\cosh(L-L_1)\beta}\right. \\ &\left. - 2\pi D_0\exp(-\delta^2T)\sum_{n=0}^{\infty}E_1E_2\right] - \frac{\mu_0}{\gamma_0}\beta\exp(-\beta L_1 + \beta\eta)\left\{\frac{\exp(-\gamma_0T)}{\beta}\frac{\sinh(\eta-L)\beta}{\cosh(L-L_1)\beta}\right. \\ &\left. - 2(L-L_1)D_0\exp(-\delta^2T)\sum_{n=0}^{\infty}(-1)^{-n}\sin\left[(n+1/2)\pi\frac{(\eta-L)}{(L-L_1)}\right]E_2\right\} + \frac{\mu_0}{\gamma_0}\beta\exp(-\beta L_1 + \beta\eta) \\ &\times\left\{E_3 - 2(L-L_1)D_0\exp(-\delta^2T)\sum_{n=0}^{\infty}(-1)^{-n}\sin\left[(n+1/2)\pi\frac{(\eta-L)}{(L-L_1)}\right]E_4\right\} \end{aligned}$$

$$\begin{aligned} F_2(\eta,T) &= \exp\left[\alpha\eta + (b-\delta^2)T\right] - \exp\left[\alpha L - \beta L + \beta\eta - \delta^2T\right] \\ &\times\left[\exp(bT)\frac{\cosh(\eta-L_1)\sqrt{b/D_0}}{\cosh(L-L_1)\sqrt{b/D_0}} - 2\pi D_0\sum_{n=0}^{\infty}E_1E_5\right] - (\alpha-\beta)\exp\left[\alpha L_1 - \beta L_1 + \beta\eta - \delta^2T\right] \\ &\times\left\{\frac{\exp(bT)}{\sqrt{b/D_0}}\frac{\sinh(\eta-L)\sqrt{b/D_0}}{\cosh(L-L_1)\sqrt{b/D_0}} - 2(L-L_1)D_0\sum_{n=0}^{\infty}(-1)^{-n}\sin\left[(n+1/2)\pi\frac{(\eta-L)}{(L-L_1)}\right]E_5\right\} \end{aligned}$$

$$F_3(\eta,T) = \exp(-\beta L + \beta\eta)E_6 - 2\pi D_0\exp\left[(-\beta L + \beta\eta) - \delta^2T\right]\sum_{n=0}^{\infty}E_1E_4$$

$$\begin{aligned} F_4(\eta,T) &= \exp(-\beta L + \beta\eta)\left[\left\{\frac{(L-L_1)}{2}\sqrt{\frac{1}{\delta^2D_0}}\tanh\left((L-L_1)\sqrt{\frac{\delta^2}{D_0}}\right) - mR_fT\right\}E_6 - \frac{mR_f(\eta-L_1)}{2}E_3\right] \\ &- 2\pi D_0\exp\left[(-\beta L + \beta\eta) - \delta^2T\right]\sum_{n=1}^{\infty}\frac{mR_f(L-L_1)^2E_1E_4}{\left[(n+1/2)^2\pi^2D_0 + \delta^2(L-L_1)^2\right]} \end{aligned}$$

$$E_1 = (-1)^{-n}(n+1/2)\cos\left[(n+1/2)\pi\frac{(\eta-L_1)}{(L-L_1)}\right],\quad E_2 = \frac{\exp\left[-\frac{(n+1/2)^2\pi^2D_0}{(L-L_1)^2}T\right]}{\left[(n+1/2)^2\pi^2D_0 + \omega^2(L-L_1)^2\right]}, \tag{11}$$

$$E_3=\sqrt{\frac{1}{\delta^2 D_0}}\frac{\sinh(\eta-L)\sqrt{\delta^2/D_0}}{\cosh(L-L_1)\sqrt{\delta^2/D_0}},\quad E_4=\frac{\exp\left(-\frac{(n+1/2)^2\pi^2 D_0}{(L-L_1)^2}T\right)}{\left[(n+1/2)^2\pi^2 D_0+\delta^2(L-L_1)^2\right]},$$

$$E_5=\frac{\exp\left[-\frac{(n+1/2)^2\pi^2 D_0}{(L-L_1)^2}T\right]}{\left[(n+1/2)^2\pi^2 D_0+b(L-L_1)^2\right]},\quad E_6=\frac{\cosh(\eta-L_1)\sqrt{\delta^2/D_0}}{\cosh(L-L_1)\sqrt{\delta^2/D_0}},$$

$$b=D_0(\alpha-\beta)^2,\ \delta^2=\gamma_0+\omega^2,\ \beta=\frac{U_0}{2D_0},\ \omega^2=\frac{U_0^2}{4D_0},\ D_0=D_{x_0}+D_{y_0},$$

$$U_0=u_0+v_0,\quad L=L_x+L_y,\ L_1=L_{x_1}+L_{y_1},\ \eta=x+y,\ T=\frac{1}{mR_f}\left[1-\cos(mt)\right]\cdot$$

4. NUMERICAL METHOD

Analytical solutions of the advection-diffusion equation have been reported for specific initial and boundary conditions. This lack of generality limits their applicability. Moreover, such solutions tend to be complex. Thus for example, analytical solutions (9) and (10) of the advection-diffusion equation (5) are clearly lengthy and contain infinite series (11). In contrast, numerical methods are generally flexible to accommodate arbitrary initial distribution and boundary conditions [18−21, 27−29]. In order to employ the EFDM to solve equation (5), this equation is first rewritten in the following form:

$$R_f\frac{\partial C(x,y,t)}{\partial t}=D_{x_0}\left|\sin(mt)\right|\frac{\partial C^2(x,y,t)}{\partial x^2}-u_0\left|\sin(mt)\right|\frac{\partial C(x,y,t)}{\partial x}$$
$$+D_{y_0}\left|\sin(mt)\right|\frac{\partial C^2(x,y,t)}{\partial y^2}-v_0\left|\sin(mt)\right|\frac{\partial C(x,y,t)}{\partial y}-\gamma_0\left|\sin(mt)\right|C+\mu_0\left|\sin(mt)\right| \tag{12}$$

The central difference scheme is used to represent the term $\left(\partial^2 C(x,t)/\partial x^2\right)$, $\left(\partial^2 C(y,t)/\partial y^2\right)$, $\left(\partial C(x,t)/\partial x\right)$ and $\left(\partial C(y,t)/\partial y\right)$ and a forward difference scheme for the derivative term $\left(\partial C(x,t)/\partial t\right)$ [27]. With these substitutions, equation (12) transforms into:

$$C_{i,j,k+1}=(G_j-E_j)C_{i+1,j,k}+(G_j+E_j)C_{i-1,j,k}+(H_j-F_j)C_{i,j+1,k}$$
$$+(F_j+H_j)C_{i,j-1,k}+(1-I_j-2H_j-2G_j)C_{i,j,k}+J_j \tag{13}$$

where indexes *i, j* and *k* refer to the discrete step lengths Δx, Δy and Δt for the coordinate *x*, coordinate *y* and time *t*, respectively, and where:

$$E_j=\frac{u_0\left|\sin(mt_j)\right|\Delta t}{2R_f\Delta x},\ F_j=\frac{v_0\left|\sin(mt_j)\right|\Delta t}{2R_f\Delta y},\ G_j=\frac{D_{x_0}\left|\sin(mt_j)\right|\Delta t}{R_f\Delta x^2},$$
$$H_j=\frac{D_{y_0}\left|\sin(mt_j)\right|\Delta t}{R_f\Delta y^2},I_j=\frac{\gamma_0\left|\sin(mt_j)\right|\Delta t}{R_f},\ J_j=\frac{\mu_0\left|\sin(mt_j)\right|\Delta t}{R_f} \tag{14}$$

The truncation error for the difference equation (13) is $O(\Delta t,\ \Delta x^2,\Delta y^2)$. Using a small-enough value of Δx, Δy and Δt the truncation error can be reduced until the accuracy achieved is within the error tolerance [30].

The initial condition (6) for equation (13) can be expressed in the finite difference form as:

$$C_{i,j,0}=C_{in}\exp(\alpha x_i+\alpha y_j),\ L_{x_1}\le x_i\le L_x;L_{y_1}\le y_j\le L_y;\ t=0 \tag{15}$$

Boundary conditions (7) and (8), rewritten in the finite difference form, are:

$$C_{N,R,k}=\begin{cases}C_0(1+\cos(mt_k)), & 0<t_k\le t_0;\ x_N=L_x,y_R=L_y\\ 0, & t_k>t_0;\quad x_N=L_x,y_R=L_y\end{cases} \tag{16}$$

$$\frac{C_{1,j,k}-C_{0,j,k}}{\Delta x}=\frac{u_0}{2D_{x_0}}C_{0,j,k},\ x=L_{x_1};\ (t\ge 0)) \tag{17}$$
$$\frac{C_{i,1,k}-C_{i,0,k}}{\Delta y}=\frac{v_0}{2D_{y_0}}C_{i,0,k},\ y=L_{y_1};\ (t\ge 0)$$

where $N=(L_x - L_{x_1})/\Delta x$ and $R=(L_y - L_{y_1})/\Delta y$ are the grid dimensions in the x and y directions, respectively. In this manner, solute concentration can be determined at different times.

5. NUMERICAL AND ANALYTICAL RESULTS

Numerical solution of Eq. (13) is obtained by EFDM over a finite domain bounded by $0 \le x \le 1$ km in the longitudinal and $0 \le y \le 1$ km in the transverse direction. To enable the comparison of results, this was done for the same set of input data previously used by Yadav et al. [24]. The input parameters are D_{x_0}=1.77 km^2/day, D_{y_0}=0.177 km^2/day, u_0=0.70 km/day, v_0=0.070 km/day, α=0.025 km^{-1}, γ_0=0.01 day^{-1}, μ_0=0.01, C_0=1.0, C_{in}=0.1 and m=0.1 day^{-1}. The elimination time of the source of the pollutant (the pulse width) is t_0=120 days. In the numerical calculations, the step lengths $\Delta x=\Delta y$=0.1 km and Δt=0.0005 days have been used to achieve the stability of the finite difference scheme. Numerical results matched the analytical solution well, with maximum deviation of 0.049 % for the longest time analyzed t=180 days.

With the pollution source active ($t \le t_0$) and for a fixed retardation factor of R_f=1.08, Figure 1 shows the concentration profiles at various moments in time.

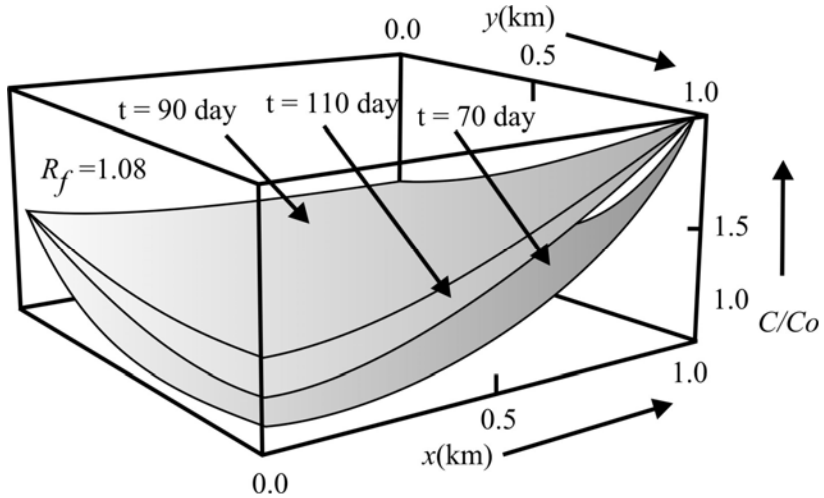

Figure 1. Solute concentration in 2D space at a varied time of t=70, 90 and 110 days (which is within the 120 days of the source activity), for a fixed retardation factor of R_f=1.08

These profiles are not in a chronological order and the uppermost surface is for the middle value of time (90 days). The lower and middle surfaces are for the times of 70 and 110 days, respectively. This is due to the periodic nature of the input concentration which is thus transported against the flow. Near the source point, the distribution of the solute in the transverse and longitudinal directions are of similar order of magnitude despite the transverse component of the velocity and dispersion coefficient being merely a tenth of the corresponding values for the longitudinal direction.

One can observe that at far end boundaries x=0 and y=0, surfaces in Fig. 1 are flat near boundaries. Concentration gradient with respect to time, at particular position, decreases. Hence, the domain $0 \le x$ and $y \le 1$ gets polluted beyond human use in a finite period of time.

For a parametric retardation factor of R_f=1.08, 1.38 and 1.68, Figure 2 shows the concentration profiles at time t=90 days.

Larger retardation factors indicate higher adsorption of the solute by the solid matrix of the porous medium. One can thus observe in Figure 2 that the concentration decreases at a higher rate for higher values of this factor R_f. It can be observed a decrease in the contaminant concentration profile with retardation factor up to a certain position, and after that position, it increases.

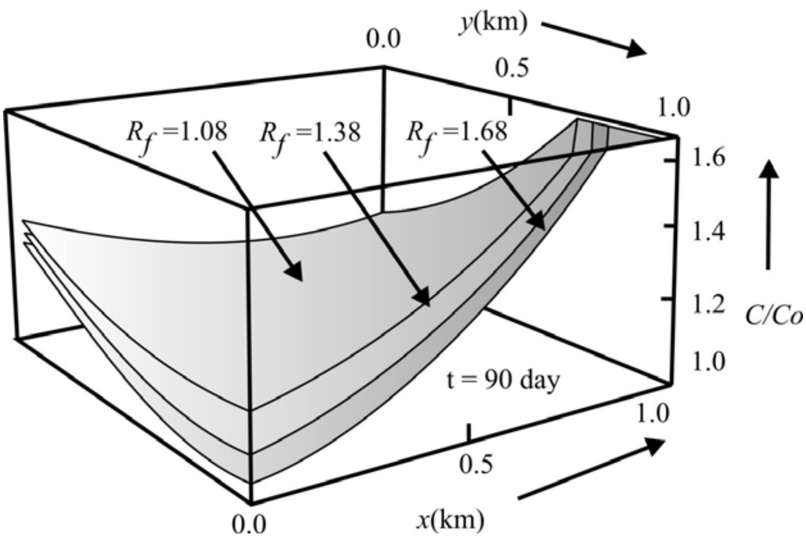

Figure 2. Solute concentration in 2D space at time t=90 days (the source is active for 120 days) for a varied value of the retardation factor of R_f=1.08, 1.38 and 1.68

Figures 3 and 4 show concentration profiles for time $t>t_0$ (the pollution source turned inactive). As the source of the pollutant is extinguished (at t_0=120 days), the concentration at the source point (x=1 km and y=1 km) becomes zero. However, the domain $0\leq x$ and $y\leq 1$ gets polluted until 120 days, so a peak concentration value occurs at source position in the domain. Figure 3 illustrates concentration profile at time t=140, 160 and 180 days and a fixed retardation factor of R_f=1.08.

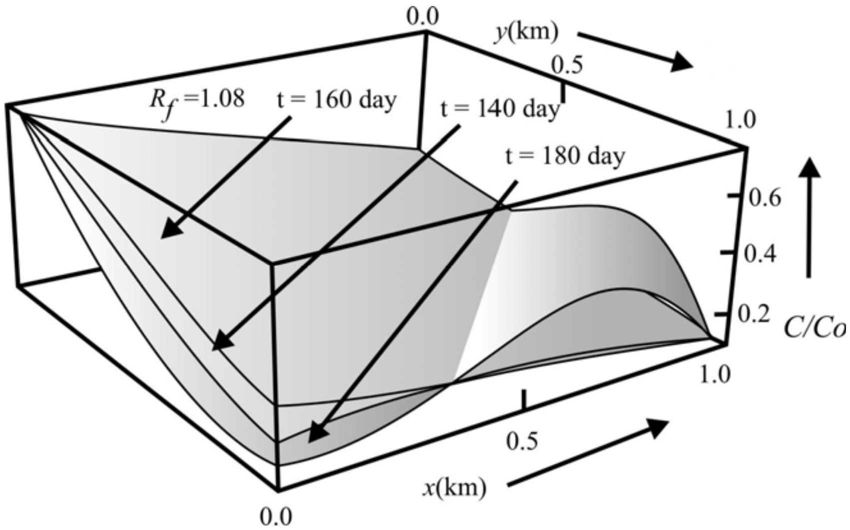

Figure 3. Solute concentration in 2D space at a varied time of t=140, 160 and 180 days (the source was active for the first 120 days), for a fixed retardation factor of R_f=1.08

One can observe in Figure 3 that this peak value lowers and drifts away from the point source with time. The rate of change in the peak concentration and concentration at some position in general are of increasing nature in time. In other words, the rehabilitation process will progress faster, aided by the transverse transport. It is also observed that the trend of contaminant concentration with time and distance travelled is almost the same as for $t\leq t_0$. The concentration values are changing periodically with time and position. The upper surface represents solute concentration at t=160 days, while middle and lower surfaces indicate lower and higher time at the same position.

Figure 4 shows the concentration profiles at time t=160 days, which is after the pollution source turned inactive ($t>t_0$). Results are shown for three values of the retardation factor: R_f =1.08, 1.38 and 1.68. The solute concentration decreases with the increasing retardation factor.

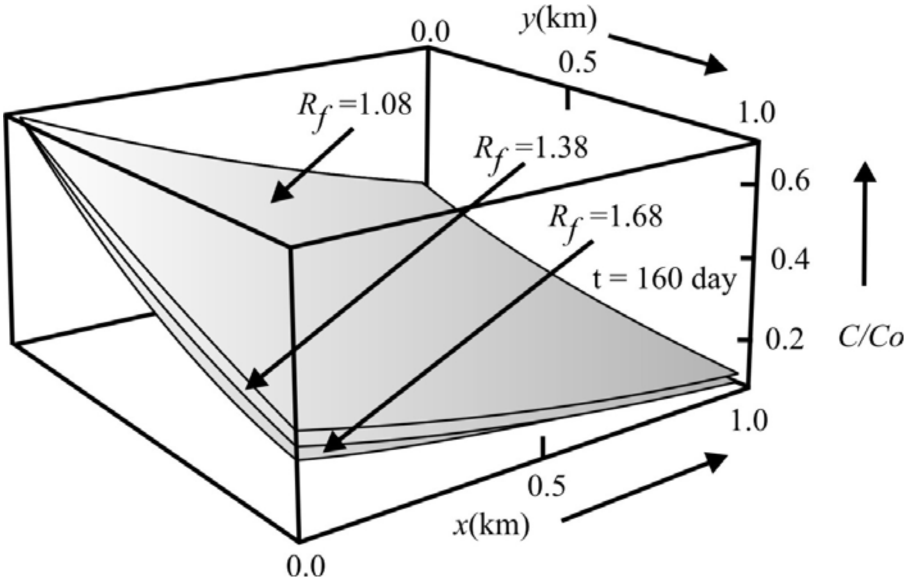

Figure 4. Solute concentration in 2D space at time t=160 days (the source was active for the first 120 days), for a varied value of the retardation factor of R_f=1.08, 1.38 and 1.68

The derived numerical solution with periodic velocity and boundary conditions are useful for understanding the transient response of ground water systems to water level changes.

Finally, the EFDM used in this work for solving two-dimensional advection-diffusion equation with variable coefficients over a finite domain can be used with arbitrary initial and boundary conditions as well as with different variations of dispersion and velocity. This is its advantage against a large number of commercial codes which can be used only for solving the advection-diffusion equation with generic boundary conditions but not for time- and/or space-dependent dispersion and velocity coefficients.

6. CONCLUSION

Solutions by the explicit finite difference method (EFDM) are given for the dispersion of pollution through a horizontal homogeneous porous medium and in presence of an unsteadiness of the dispersion and velocity that is of periodic nature. Accounted is the retardation that occurs in the porous medium due to adsorption. Numerical results are compared to analytical solutions reported in the literature and good agreement is apparent, with maximum deviation of 0.049 % for the longest time analyzed t=180 days. It was found that the solute concentration profile is influenced significantly by the periodic velocity fluctuations. The proposed numerical solution is a useful tool in trying to understand the physical mechanism of the groundwater flow in porous media. As a modeling tool, the proposed numerical solution of advection–dispersion equation was also shown to be accurate.

Finally, the EFDM used in this work is effective and accurate for solving the two-dimensional advection-diffusion equation with variable coefficients over a finite domain. Such numerical method for solving two-dimensional advection-diffusion equation with variable coefficients over a finite domain can be used with arbitrary initial and boundary conditions as well as with different variations of dispersion and velocity. This is of special importance when an analytical solution is not available, or it is lengthy and contains infinite series.

7. ACKNOWLEDGMENT

The work described in this paper was supported by the Strategic Research Grant of City University of Hong Kong (Project No. CityU 7004069) and by a grant from Serbian Ministry of Education, Science and Technological Development (Project No. 171011).

8. REFERENCES

[1] C. S. Rao, *Environmental Pollution Control Engineering*, 3rd reprint, Wiley Eastern Ltd, New Delhi, 1995.

[2] F. T. Lindstrom, L. Boersma, *Analytical solutions for convective-dispersive transport in confined aquifers with different initial and boundary conditions*, Water Resources Res., Vol. 15 (1989) 241–256.

[3] J. D. Logan, V. Zlotnik, *The convection–diffusion equation with periodic boundary conditions*, Applied Mathematics Letters, Vol. 8 (1995) 55–61.

[4] L. R. Townley, *The response of aquifers to periodic forcing*, Advances in water Resources, Vol. 18 (1995) 125–146.

[5] D. K. Jaiswal, A. Kumar, N. Kumar, R. R. Yadav, *Analytical solutions for temporally and spatially dependent solute dispersion of pulse type input concentration in one-dimensional semi-infinite medium*, Journal of Hydro-Environment Research, Vol. 2 (2009) 254–263.

[6] D. K. Jaiswal, A. Kumar, R. R. Yadav, *Analytical solution to the one-dimensional advection-diffusion equation with temporally dependent coefficients*, Journal of Water Resource and Protection, Vol. 3 (2011) 76–84.

[7] A. Kumar, D. K. Jaiswal, N. Kumar, *Analytical solutions to one-dimensional advection-diffusion equation with variable coefficients in semi-infinite media*, J. Hydrology, 380 (2010) 330–337.

[8] R. R. Yadav, D. K. Jaiswal, H. K. Yadav, Gulrana, *Temporally dependent dispersion through semi-infinite homogeneous porous media: an analytical solution*, International Journal of Research and Reviews in Applied Sciences, Vol. 6 (2010) 158–164.

[9] E. Ciftci, C. B. Avci, O. S. Borekci, A. U. Sahin, *Assessment of advective-dispersive contaminant transport in heterogeneous aquifers using a meshless method*, Environmental Earth Sciences, Vol. 67 (2012) 2399–2409**.**

[10] G. A. Assumaning, S. -Y. Chang, *Use of simulation filters in three-dimensional groundwater contaminant transport modeling*, J. Environmental Eng. (US), Vol. 138 (2012) 1122−1129.

[11] S. Dhawan, S. Kapoor, S. Kumar, *Numerical method for advection-diffusion equation using FEM and B-splines*, J. Computational Science, Vol. 3 (2012) 429–437.

[12] M. Dehghan, *Weighted finite difference techniques for the one-dimensional advection-diffusion equation*, Appl. Math. Computation, Vol. 147 (2004) 307–319.

[13] H. Karahan, *Implicit finite difference techniques for the advection–diffusion equation using spreadsheets*, Adv. Eng. Software, Vol. 37 (2006) 601–608.

[14] Q. Huang, G. Huang, H. Zhan, *A finite element solution for the fractional advection-dispersion equation*, Adv. Water Resources, Vol. 31 (2008) 1578–1589.

[15] C. Zhao, S. Valliappan, *Transient infinite element for contaminant transport problems*, Int. J. Num. Meth. Eng., Vol. 37 (1994) 113–1158.

[16] M. Srećković, A. A. Ionin, A. Janićijević, A. Bugarinović, S. Ostojić, M. Janićijević, N. Ratković Kovačević, *Formalisms analysis, results and accomplishments with population inversion of material*, Contemporary Materials, Vol. VIII−1 (2017) 91−108.

[17] C. Zhao, S. Valliappan, *Numerical modelling of transient contaminant migration problems in infinite porous fractured media using finite/infinite element technique: theory*, Int. J. Num. Anal. Meth. Geomech., Vol. 18 (1994) 523–541.

[18] S. Savović, A. Djordjevich, *Finite difference solution of the one-dimensional advection-diffusion equation with variable coefficients in semi-infinite media*, Int. J. Heat Mass Transfer, Vol. 55 (2012) 4291–4294.

[19] S. Savović, A. Simović, A. Djordjevich, A. Janićijević, *Equilibrium mode distribution in W-type glass optical fibers*, Contemporary Materials, Vol. V−1 (2014) 51−58.

[20] S. Savović, A. Djordjevich, *Numerical solution for temporally and spatially dependent solute dispersion of pulse type input concentration in semi-infinite media*, Int. J. Heat Mass Transfer, Vol. 60 (2013) 291–295.

[21] A. Djordjevich, S. Savović, *Solute transport with longitudinal and transverse diffusion in temporally and spatially dependent flow from a pulse type source*, International Journal of Heat and Mass Transfer, Vol. 65 (2013) 321−326.

[22] A. N. S. Al-Niami, K. R. Rushton, *Analysis of flow against dispersion in porous media*, Journal of Hydrology, Vol. 33 (1977) 87–97.

[23] M. A. Marino, *Flow against dispersion in non-adsorbing porous media*, Journal of Hydrology, Vol. 37 (1978) 149–158.

[24] R. R. Yadav, D. K. Jaiswail, G. Gulrana, *Two-dimensional solute transport for periodic flow in isotropic porous media: an analytical solution*, Hydrol. Process., Vol. 26 (2012) 3425–3433.

[25] L. Lapidus, N. R. Amundson, *Mathematics of adsorption in beds*, VI. The effects of longitudinal diffusion in ion-exchange and chromatographic columns, Vol. 56 (1952) 984–988.

[26] J. A. Cherry, R. W. Gillham, J. F. Barker, *Contaminants in groundwater- Chemical processes in Groundwater contamination*, National Academy Press: Washington, DC (1984) 46–64.

[27] S. Savović, J. Caldwell, *Finite difference solution of one-dimensional Stefan problem with periodic boundary conditions*, Int. J. Heat Mass Transfer, Vol. 46 (2003) 2911–2916.

[28] A. Janićijević, S. Savović, A. Djordjevich, *Numerical solution of the diffusion for oxygen diffusion in soil*, Contemporary Materials , Vol. VII−1 (2016) 6−10.

[29] S. Savović, J. Caldwell, *Numerical solution of Stefan problem with time-dependent boundary conditions by variable space grid method*, Thermal Sci., Vol. 13 (2009) 165–174.

[30] J. D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, New York, 1995.

РЕШЕЊЕ КОНАЧНИХ ДИФЕРЕНЦИЈА ДВОДИМЕНЗИОНАЛНОГ ТРАНСПОРТА РАСТВОРЕНЕ СУПСТАНЦЕ СА ПЕРИОДИЧНИМ ТОКОМ У ХОМОГЕНОМ ПОРОЗНОМ МЕДИЈУ

Сажетак: Дводимензионална адвекцијско-дифузиона једначина са варијабилним коефицијентима решена је експлицитним методом коначних диференција за транспорт растворених супстанци кроз хомогени, коначни, порозни, дводимензионални домен. Дозвољени су ретардација адсорпцијом, периодична брзина филтрације и коефицијент дисперзије пропорционалан овој брзини. Транспорт се врши из пулсног стационарног извора (који нестаје након периода активности). Укључени су распад првог реда и параметри производње нултог реда пропорционални брзини филтрације, периодичним граничним условима на почетку и на крају домена. Резултати се пореде са аналитичким решењима наведеним у литератури за специјалне случајеве и постигнуто је добро слагање. На профил концентрације растворене супстанце увелико утичу периодичне флуктуације брзине. Решења за мноштво комбинација непостојаности коефицијената у адвекцијско-дифузионим једначинама могу се добити као појединачни случајеви оног који је овдје приказан. Ово даље доказује делотворност експлицитног метода коначне диференције за решење дводимензионалне адвекцијско-дифузионе једначине са варијабилним коефицијентима у коначном медију, што је посебно важно када су потребни произвољни иницијални и гранични услови.

Кључне речи: дводимензионална адвекцијско-дифузиона једначина, пренос масе, метод коначне диференције.