

Fiber shape reconstruction with fractal nature analysis

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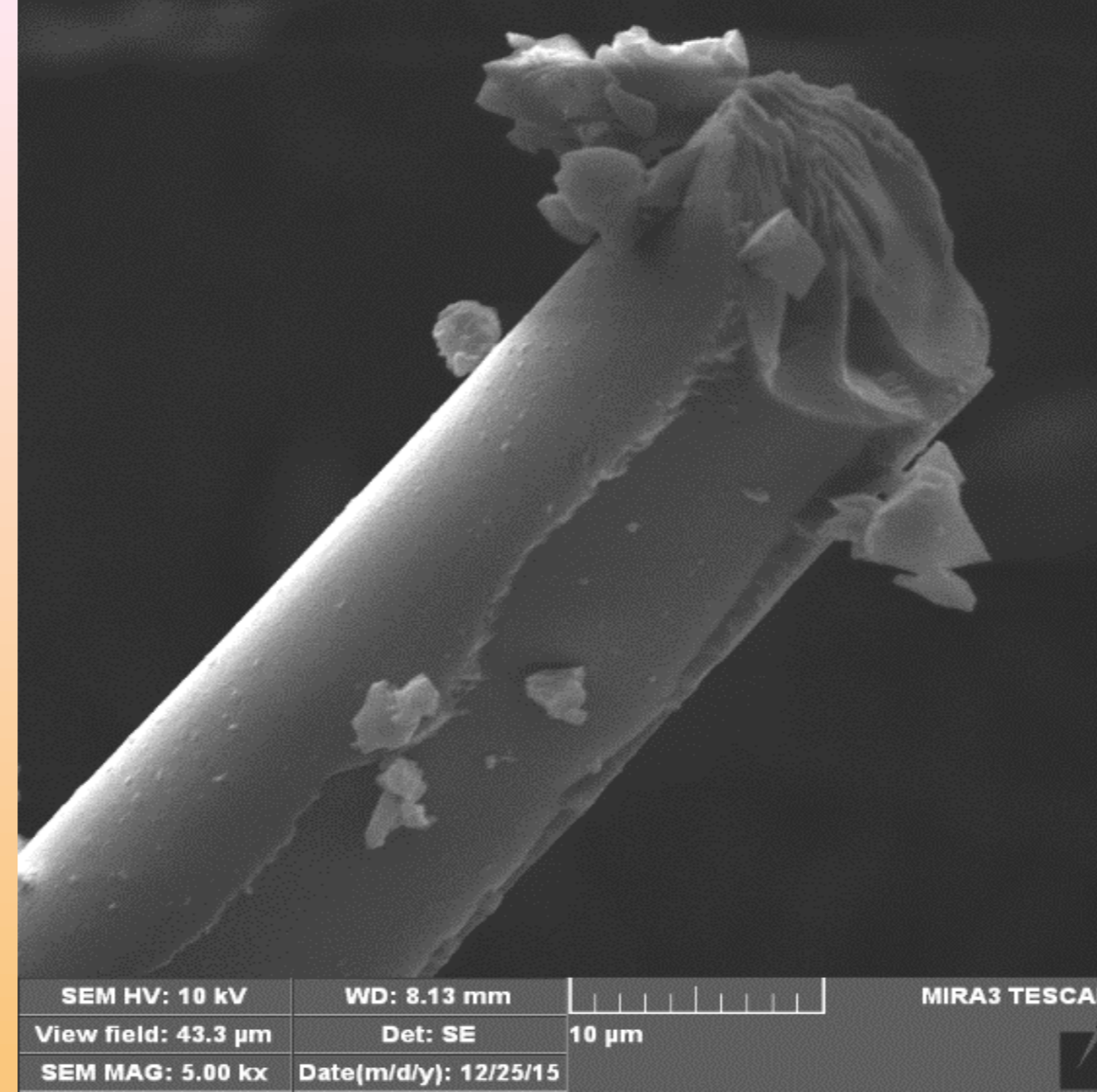
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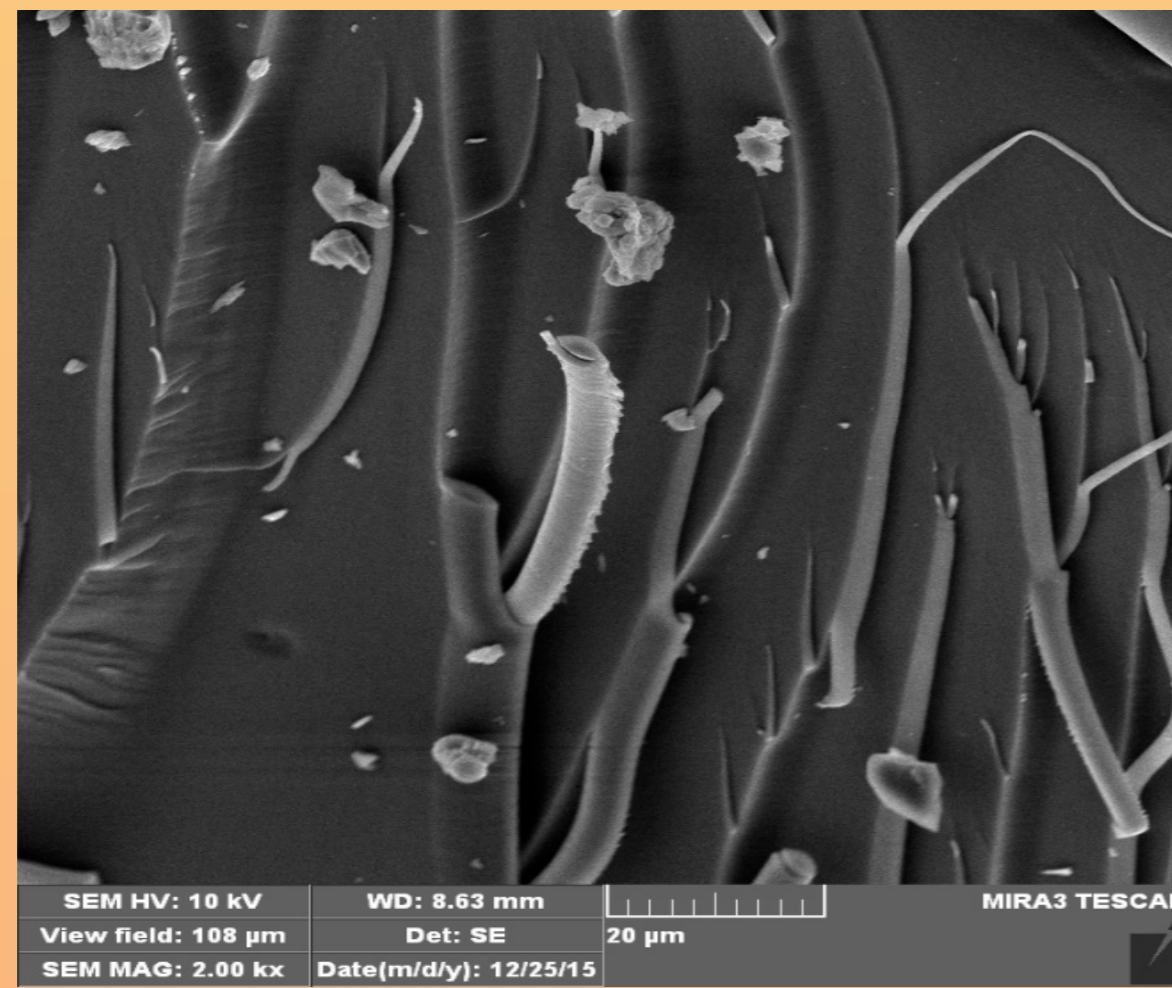
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ABSTRACT

Polymer composites represent very useful materials due to a possibility to design various properties with the combination of ceramic reinforcements in the shape of particles and fibers. Thanks to this, composites are being applied in every industry, from commercial products to high performance materials for aircrafts. Microelectronic devices contain polymers and composites as insulators and adhesives, mostly epoxy-based. In order to improve their mechanical properties, such as modulus of elasticity of impact resistance, and endurance, glass fibers are incorporated as reinforcement. For the design of a composite with desired mechanical properties and long-life, thorough knowledge of microstructural changes and fiber-matrix interface is required. Microstructural analysis can also provide insight in reinforcement shape that can enable connection with the physical properties. Fractal nature analysis represents mathematical tool that can be used for the shape and size reconstruction, ensuring prediction of different properties, which is valuable for the future materials processing. In this paper, field emission scanning electron microscopy (FESEM) images were used for fibers microstructure fractal analysis. Reconstruction of fiber shape was obtained successfully, opening the door for the application of fractal analysis shape reconstruction application on other materials, with the future focus on nanomaterials used in microelectronic devices.



FESEM of a broken glass fiber.



FESEM of a fiber-reinforced composite

INTRODUCTION

The use of polymers and composites with a polymer matrix has reached every field of application, from robust constructions to fine, delicate parts for microelectronic devices [1]. Reinforcement with fibers that resulted in fiber-reinforced composites (FRC) brought improved impact strength, modulus of elasticity and toughness to polymer matrix [2]. However, under the influence of load, FRC are subject to the formation of cracks, separation and stratification of fibers from the matrix [3-5]. With thorough knowledge of the FRC structure, physical properties can be predicted and included in the processing of future composites, especially that electronic materials miniaturization brought micro- and nanoscale level properties at spotlight. The fractal nature exists within physical systems structures and contact surfaces, from microstructures, down to the nano-scale level, up to the global bulk and massive shapes. Fractal nature analysis presents a possible approach for the investigation of contact phenomena establishing the grain contacts models, offering ceramics and other materials structure analysis, description and prediction of grains' and pores shape, along with relations between structure and electric-dielectric properties. This mathematical technique can be performed on field emission scanning electron microscopy (FESEM) images, by identifying fiber phase and pores shapes and boundaries, as well as fiber-matrix bonding at the interface. In this study, fiberglass mat was used for the reinforcement of epoxy. FESEM image of enlarged fiber after the composite fracture was used for the reconstruction of data.

FRACTAL RECONSTRUCTION OF DATA

Fractal nature analysis of experimentally determined physical properties is performed using a novel affine fractal regression model described by the equations published in our previous research. The aim is to find coefficients that fit experimental data for the following equation system:

$$\varphi\left(\frac{x+j}{p}\right) = a_j \varphi(x) + b_j x + c_j,$$

where $x \in [0,1)$, $0 \leq j \leq p-1$, a_j represent fractal and b_j directional coefficients, with $0 < |a_j| < 1$, with domain $[0,1)$, p stands for fractal period. Real solution equation system is called fractal function $\varphi: [0,1) \rightarrow \mathbb{R}$, having mathematical fractal structure – function graph plot represents fractal curve. Higher a_j appear in the case of strong fractal oscillations. The curve fractal level defined by the equation system is L ; the first fractal level is replicated in the entire domain over every of the p sub-intervals, building the second fractal level. In order to obtain coefficients that fit the data, explicit solution of the problem that depends on the p -expansion of numbers in $[0,1)$ is used.

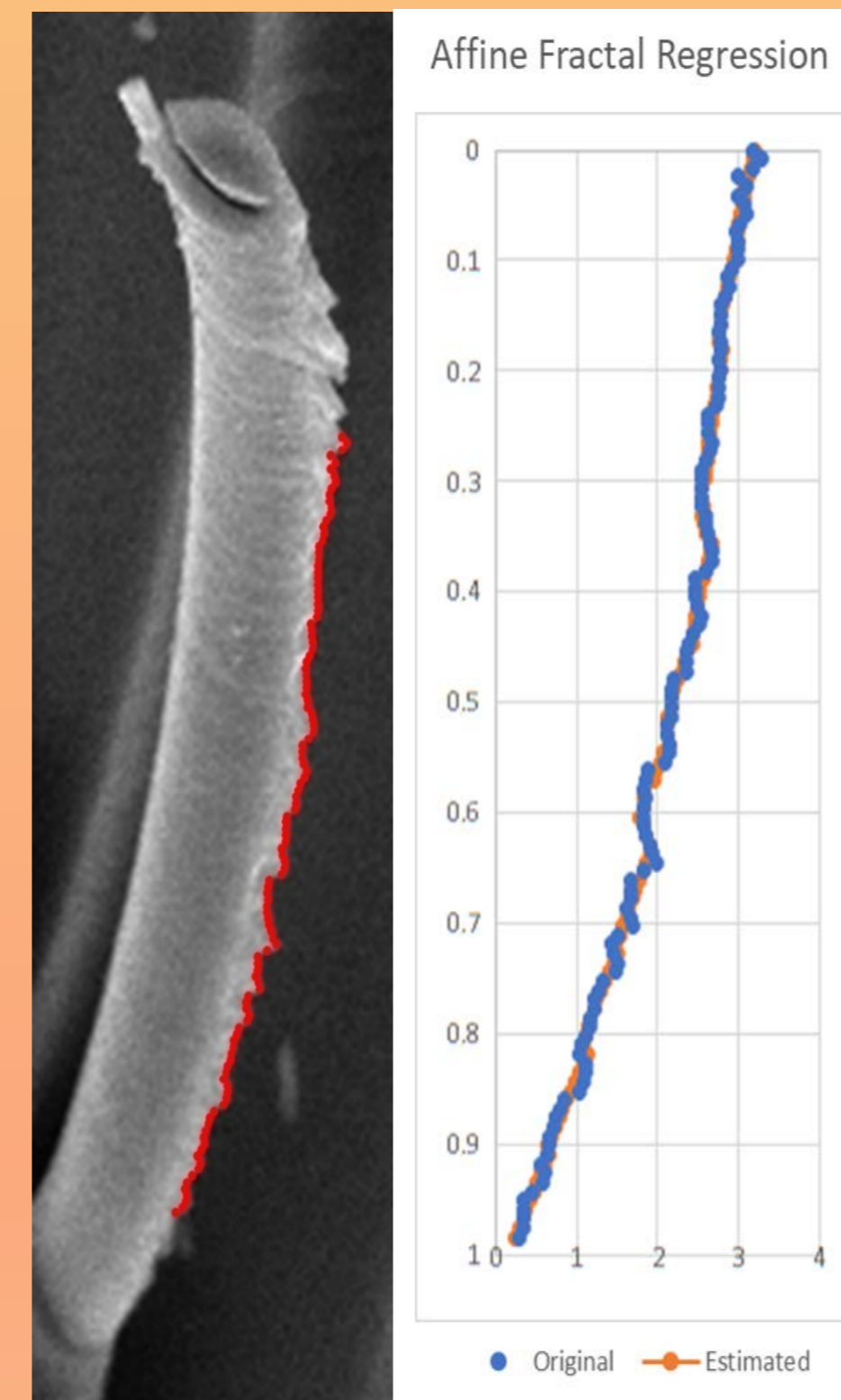
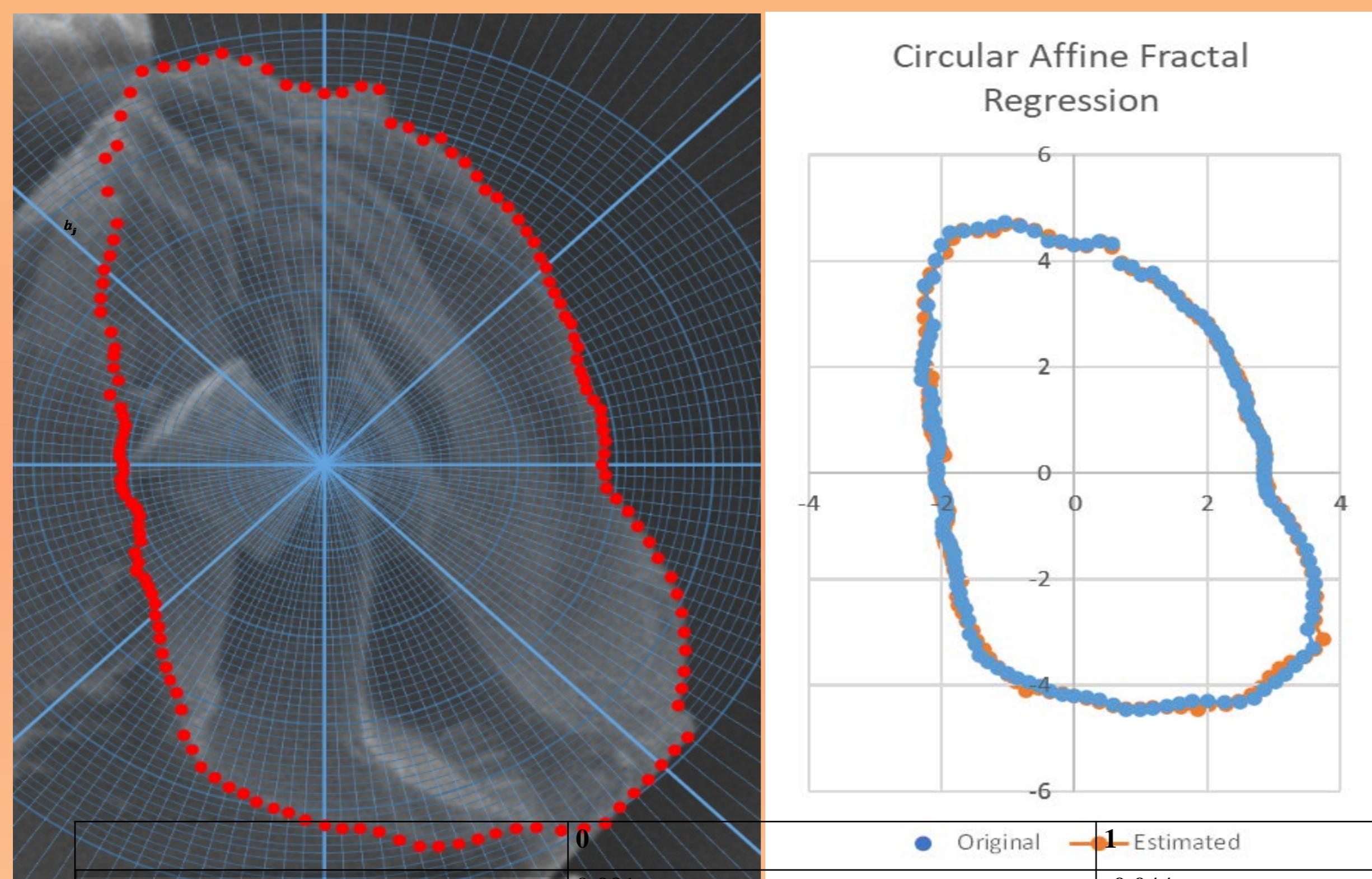


Image of a fiber with red points (on left) and corresponding fractal reconstruction (on right)

TABLE ESTIMATED COEFFICIENTS FOR THE FRACTAL CURVE OF THE RADIUS

	0	1	2	3	4	5
a_j	-0.058	-0.086	-0.009	-0.229	-0.076	-0.161
b_j	-0.397	-0.47	-0.224	-0.606	-0.446	-0.701
c_j	3.39	3.256	2.849	3.371	2.896	2.886
	6	7	8	9	10	
a_j	-0.365	-0.021	-0.097	0.001	-0.017	
b_j	-1.2	-0.518	-0.725	-0.561	-0.54	
c_j	3.244	1.976	-0.017	7.809	1.135	0.703
	11	12	13	14	15	16
a_j	0.027	-0.057	-0.013			
b_j	0.695	-0.005	-1.654			
c_j	3.957	5.072	4.608	2.821	2.868	3.304

TABLE Enlarged tip of the broken fiber and fractal curve of the fiber shape

TABLE ESTIMATED COEFFICIENTS FOR THE FRACTAL CURVE OF THE RADIUS

	0	1	2	3	4	5
a_j	0.081	-0.044	0.012	-0.002		
b_j	0.731	-2.212	-0.805	-0.098	0.603	1.362
c_j	3.951	4.989	2.695	2.118	2.156	2.852
	6	7	8	9	10	11
a_j	0.027	-0.057	-0.013	0.021	-0.013	0.024
b_j	0.695	-0.005	-1.654	-0.037	0.517	0.712
c_j	3.957	5.072	4.608	2.821	2.868	3.304

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THE CONCLUSION

In this paper, fractal nature analysis was applied on fiber-reinforced composite for the reconstruction of fiber shape. The analysis with software Fractal Real Finder, fractal curve depicting the shape was obtained, as well as Hausdorff dimension of 1 and 1.21968, for the first and the second analysis. This indicates that the fibers have been successfully reconstructed. The finding achieved in this study enables the use of the fractal software analysis for the design and prediction of efficient reinforcement for epoxy-based composites in the future.